ALL ANSWERS MUST BE GIVEN ON THE ANSWER SHEET
BY CROSSING THE CORRESPONDING LETTER

PART I: MATHEMATICS M.C.Q's

Number of Math M.C. Questions: 50 | Time Allowed: 100 minutes
Questions on Page Numbers 1 to 7 | Negative Marking: Yes

1. Suppose that $f(x) = \ln(x)$ and $g(x) = 9 - x^2$. The domain of $f(g(x))$ is

   A) $|x| \geq 3$ B) $|x| \leq 3$ C) $|x| > 3$ D) $|x| < 3$.

2. Let A be a set. What does it mean for A to be uncountable?

   A) There is no way to assign a distinct element of A to each natural number.
   B) There exist elements of A which cannot be assigned to any natural number at all.
   C) There is no way to assign a distinct natural number to each element of A.
   D) There is a bijection f from A to the real numbers $\mathbb{R}$.

3. The graph of $y^2 = x^2 + 9$ is symmetric with respect to
   (I) the x-axis.
   (II) the y-axis.
   (III) the origin.
   A) I only.
   B) III only.
   C) II and III.
   D) I, II and III.

4. If $f : \mathbb{R} \to \{-1, 1\}$ be onto, then
   A) f is not continuous.
   B) f is continuous.
   C) f is differentiable everywhere.
   D) f is continuous, but not differentiable anywhere.

5. If the amplitude of $y = (1/k)\cos(k^2\theta)$ is 2,then its period must be

   A) π B) 2π C) 4π D) 8π.

6. If $[x]$ denotes the greatest integer $\leq x$, then $\lim_{x \to 1/2} [x] =$

   A) 0 B) $1/2$ C) 1 D) The limits does not exist.

7. Which of the following is or are true?
   (I) $\lim_{x \to 2} x^2 + 2x - 1 = 7$
   (II) $\lim_{x \to -3} \dfrac{x^2 + 5x + 6}{x^2 - x - 12} = \dfrac{1}{7}$
   (III) $\lim_{x \to 9} \dfrac{3 - \sqrt{x}}{9 - x} = +\infty$
   A) I only.
   B) I and II only.
   C) I and III.
   D) I, II and III .

8. The graph of the following are asymptotic to the x-axis EXCEPT

   A) $y(x^2 + 1) = 4x$
   B) $y = e^{x-2}$
   C) $xy = 1$
   D) $y = -\ln(x + 1)$

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9. The function $f(x) = \dfrac{x^2+5x+6}{x^2-4}$ has
   A) only a removable discontinuity at $x = -2$
   B) removable discontinuities at $x = 2$ and $x = -2$
   C) a removable discontinuity at $x = -2$ and a nonremovable discontinuity at $x = 2$
   D) a removable discontinuity at $x = 2$ and a nonremovable discontinuity at $x = -2$

10. If $f'(a)$ does NOT exist, which of the following MUST be true?
   A) $f(x)$ is discontinuous at $x = a$.
   B) f has a vertical tangent at $x = a$.
   C) f has a hole at $x = a$.
   D) None of these is necessarily true.

11. If $y = x^{(x^3)}$, then $\dfrac{dy}{dx}$

   A) $x^{(x^3+2)}(1+3\ln(x))$ B) $x^{(x^3+2)}$ C) $4x^{(x^3+2)}$ D) $x^{(x^3)}(1+3\ln(x))$

12. A particle moving along a horizontal path such that its position at any time t is given by $s(t) = (2t-3)^3$. The number of times particle changes its direction is

   A) 0 B) 1 C) 2 D) not determinable from the given information

13. Let f be a real valued function whose inverse is given by the equation $f^{-1}(x) = x(1+x^2)+(1-x^2)$. What's the value of $f(f^{-1}(f(2)))$?

   A) -2 B) -1 C) 1 D) 2

14. If the roots of the equation $x^2 + Bx + 1 = 0$ are the squares of the roots of the equation $x^2 + bx + 1 = 0$, which of the following express B in terms of b?

   A) $2 - b^2$ B) $1 - b^2$ C) $b^2 - 1$ D) $b^2 - 2$

15. Let x be a real number such that $\sin(\sin(x)) = 1/2$ and $2 < x < 3$. What's the value of $\cos(-\sin(x))$?

   A) $-\sqrt{1-(\frac{\pi}{2})^2}$ B) $-\sqrt{3}/2$ C) $\sqrt{1-(\frac{\pi}{2})^2}$ D) $\sqrt{3}/2$

16. Let $\{x_n\}$ be a sequence with $x_1 = 2$ and $x_n = \sqrt{5x_{n-1}+6}$ for every integer $n \geq 2$. Given that this sequence converges, what is its limit?

   A) 4 B) 6 C) 8 D) 10

17. If $[x]$ denotes the greatest integer $\leq x$, then $\int_0^{7/2} [x]\,dx =$

   A) 5/2 B) 7/2 C) 9/2 D) 17/2

18. Evaluate this limit:
$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left[\frac{k}{n^2} - \frac{k^2}{n^3}\right] = ?$$

   A) 2/3 B) 1/2 C) 1/3 D) 1/6

19. If L is the line through the point $A = (3,2,1)$ and parallel to the vector $v = (-2,1,3)$, what's the equation of the plane that contains L and the point $B = (-2,3,1)$?
   A) $-x + y + z = 6$ B) $3x - 2y - z = 4$ C) $x + 6y - 11z = 5$ D) $x + 5y - z = 12$

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20. Which of the following is normal to the surface $\ln(x+y^2-z^3) = x-1$ at the point where $y=8$ and $z=4$?
A) $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ B) $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ C) $\mathbf{i}+2\mathbf{j}$ D) $\mathbf{j}-3\mathbf{k}$

21. Let $g(x,y)$ be the function defined for all x and all nonzero y such that the differential equation

$$(\sin(xy))dx + g(x,y)dy$$

is exact and $g(0,y)=0)$ for all $y \neq 0$. What is $g(x,1) =$?

A) $\sin(x)+\cos(x)-1$
B) $x\sin(x)+\cos(x)-1$
C) $x\cos(x)+\sin(x)-1$
D) $x\sin(x)-\cos(x)-1$.

22. A bacterial culture is growing at a rate proportional to the number of bacteria at any time t. Initially, there are $20,000$ bacteria present, and this population doubles in 3 hours. Which of the following equation describes this growth?

A) $y = 20,000e^{(\ln(2)/3)t}$
B) $y = 20,000e^{(\ln(2/3))t}$
C) $y = 20,000e^{(3\ln(1/2))t}$
D) $y = 20,000e^{(2\ln(3))t}$

23. If $F(x) = \int_1^x \sqrt{t^2+3t}\,dt$, then $F'(x) =$
A) $(x^2+3x)^{3/2}$
B) $\sqrt{x^2+3x}$
C) $\sqrt{x^2+3x}-1$
D) $(x^2+3x)^{3/2}-1$

24. Determine whether the integral

$$\int_1^3 \frac{dx}{\sqrt{x-1}}$$

converges or diverges. If it is converges, find the value to which it converges.
A) diverges B) converges to 0 C) converges to 1 D) converges to $2\sqrt{2}$

25. What are the values of x for which the series

$$\sum_{n=0}^{+\infty} \frac{x^{3n+1}}{(3n+1)!}$$

converges?

A) converges for all x
B) $|x| < 3$
C) only at $x=0$
D) series diverges for all x

26. If both 11^2 and 3^3 are factors of the number $a \cdot 4^3 \cdot 6^2 \cdot 13^{11}$, then what is the smallest possible value of a?
A) 121 B) 3267 C) 363 D) 33

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27. Which of the following series converge?

(I) $\sum_{n=1}^{+\infty} \frac{3}{n}$

(II) $\sum_{n=1}^{+\infty} \frac{n+1}{n+4}$

(III) $\sum_{n=1}^{+\infty} \frac{-2}{(-5)^n}$

A) I only
B) I and II only
C) I and III
D) III only

28. A power series for $\sin(x^2)$ could be:

A) $1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \frac{x^{16}}{8!} \dots$

B) $x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \frac{x^{18}}{9} \dots$

C) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \frac{x^{18}}{9!} \dots$

D) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} \dots$

29. A point P moves so that the product of its distances from two fixed points Q and Q' is a^2. If the polar coordinates of Q and Q' are $(a, 0)$ and (a, π) respectively, find the polar equation of the locus.
A) $r = 2a\cos(\theta)$
B) $r^2 = 2a^2\cos(2\theta)$
C) $r^2 = 2a^2\sin(2\theta)$
D) $r = 2a\sin(\theta)$

30. Choose values of the real constants b and c so that the function $w = (x + 2y) + i(bx + cy)$ becomes an analytic function of $z = x + iy$.
A) $c = 2, b = 1$ B) $c = -2, b = 1$ C) $c = 1, b = -2$ D) $c = -2, b = 1$

31. Topology deals mainly with these properties of configurations which are invariant under

A) conformal mapping
B) continuous, one-to-one transformation
C) Euclidean transformation
D) contact transformation

32. The set of 2×2 matrices fails to satisfy the requirement of a group under multiplication because

A) the closure law is not satisfied
B) the set lacks an identity element
C) the associative law is not satisfied
D) not every element has an inverse

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33. For what value of K, is $4x^2 + 8xy + Ky^2 = 9$ the equation of a pair of straight lines?

A) 0 B) 1 C) 2 D) 4

34. Consider the system of linear equations:

$$3x + 2y - 5z = 3$$
$$2x - 6y + Kz = 9$$
$$5x - 4y - z = 5$$

If this system is inconsistent, find K?

A) 2 B) -4 C) 4 D) 3

35. If the probabilities that A and B will die within a year are p and q, respectively, what is the probability that only one of them will be alive at the end of the year?

A) $p + q + pq$
B) $p + q - 2pq$
C) $p + q - pq$
D) $p + q + 2pq$

36. Let $\mathcal{P}_n(\mathbb{R})$ be the vector space of polynomials over $\mathbb{R}$ of degree n or less and $T : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ be the linear map such that $T(f(x)) = 2f(x) + 3\int_0^x f(t)\,dt$. The rank of T is

A) 1 B) 2 C) 3 D) 4

37. The sum of eigenvalues of $\begin{pmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{pmatrix}$ is

A) -3 B) -1 C) 3 D) 1

38. A set of linear equations is represented by the matrix equation $AX = b$. The necessary condition for the existence of a solution of this system is

A) A must be invertible
B) b must be linearly depended on the columns of A
C) b must not be linearly depended on the columns of A
D) None of these

39. Let A, B and C be real 2×2 matrices, and let $\mathbf{0}$ denote the 2×2 zero matrix. Which of the following statement is/are true?
(I) $A^2 = \mathbf{0} \Rightarrow A = \mathbf{0}$
(II) $AB = AC \Rightarrow B = C$
(III) A is invertible and $A = A^{-1} \Rightarrow A = I$ or $A = -I$
A) I only
B) I and III only
C) II and III only
D) None of the above

40. Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation whose kernel is a 3-dimensional subspace of $\mathbb{R}^5$. The set $\{T(\mathbf{x}) : \mathbf{x} \in \mathbb{R}^5\}$ is

A) The trivial subspace
B) a line through the origin
C) a plane through the origin
D) all of $\mathbb{R}^3$

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41. If V_1 and V_2 are 6 dimensional subspaces of a 10 dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

A) 0 B) 1 C) 2 D) 4

42. If A is 3×3 matrix such that $A \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $A \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then the product $A \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}$ is

A) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ B) $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ C) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ D) $\begin{pmatrix} 9 \\ 10 \\ 11 \end{pmatrix}$

43. Which of the following group is cyclic?

A) $\mathbb{Z}_2 \times \mathbb{Z}_4$ B) $\mathbb{Z}_2 \times \mathbb{Z}_6$ C) $\mathbb{Z}_3 \times \mathbb{Z}_4$ D) $\mathbb{Z}_3 \times \mathbb{Z}_6$

44. Which of the following rings are integral domains?
(I) $\mathbb{Z} \oplus \mathbb{Z}$
(II) $\mathbb{Z}_p$, where p is a prime
(III) $\mathbb{Z}_{p^i}$, where p is a prime
A) I and II only
B) II only
C) II and III
D) III only

45. Which of the following statements is true:

A) A number is rational if and only if its square is rational.
B) An integer n is odd if and only if $n^2 + 2n$ is odd.
C) A number is irrational if and only if its square is irrational.
D) A number n is odd if and only if $n(n+1)$ is even.

46. Consider the statement: If n is divisible by 30 then n is divisible by 2 and by 3 and by 5. Which of the following statements is equivalent to this statement?

A) If n is not divisible by 30 then n is divisible by 2 or divisible by 3 or divisible by 5.
B) If n is not divisible by 30 then n is divisible by 2 or divisible by 3 or divisible by 5.
C) If n is divisible by 2 and divisible by 3 and divisible by 5 then n is divisible by 30.
D) If n is not divisible by 2 or not divisible by 3 or not divisible by 5 then n is not divisible by 30.

47. Let $\omega = e^{2\pi i/5}$ be a fifth root of 1. What is the value of the function $f(z) = z^2 + z$ at $z = \omega + \omega^{-1}$?

A) -2 B) -1 C) 1 D) 2

48. If the variance of x is σ^2, what is the variance of $ax + b$, where a and b are constants.
A) σ^2
B) $a\sigma^2$
C) $a^2\sigma^2 + b^2$
D) $a^2\sigma^2$

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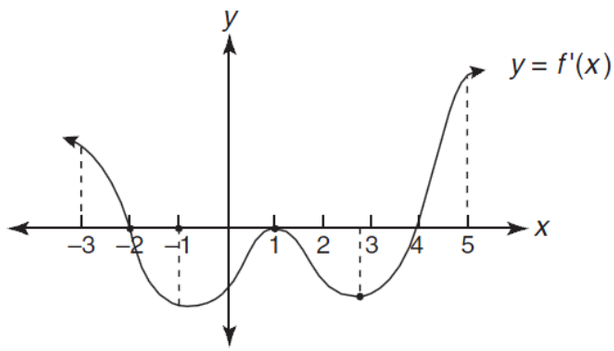
49. Let $f : [2, 4] \to \mathbb{R}$ be a continuous function such that $f(2) = 3$ and $f(4) = 6$. The most we can say about the set f([2,4]) is that
A) It is a set which contains $[3, 6]$.
B) It is a closed interval.
C) It is a set which contains 3 and 6.
D) It is a closed interval which contains $[3, 6]$.

50. Below is the graph of $f'(x)$. On what interval(s) is the graph of $f(x)$ concave upwards?



A) $(-3, 1)$ and $(1, 3)$ B) $(-4, 1)$ and $(1, 3)$ C) $(-1, 3)$ D) $(-1, 1)$ and $(3, 5)$

Congratulations ! You've finished math MCQs.
Please cross (×) Option E in the answer sheet corresponding to all unanswered MCQs.

(Stop. Do not turn over the next page until you are told to do so.)